FINANCIAL EQUILIBRIUM EQUATION IN THE ANALYSIS OF PARAMETRIC CHANGES OF A PAY AS YOU GO SCHEME

Luis Camacho
Asesoría General en Seguridad Social
Banco de Previsión Social
Uruguay

INTRODUCTION

In this study we consider certain common factors present in the individual and collective financial equations[1] of defined benefit schemes. These factors have enabled us to draw conclusions on the main differences among pay-as-you go[2], partially funded[3] and fully funded[4] schemes. Furthermore, this approach has enabled us to compare the outcomes of the different financial schemes through the analysis of the associated technical interest rates[5].

The basic factors - contribution and retirement average periods, as well as contribution and retirement central ages- applied to the different financial equilibrium equations, allow us to infer similarities between their basic formulations, only distinguishable by the magnitude of the corresponding technical interest rates.

The most significant hypothesis on which models are based, consider death as the only cause for removal from the scheme and a single age for labor force entry and retirement. In order to carry out a comparative study we assume the permanence of the mortality rate along time. This assumption is quite deficient as mortality improves constantly .

A similar methodology is used for further development, but not considering this time the improvement assumption regarding future mortality. This procedure enables us to formulate certain general observations on possible parametric reforms to the pay as you go scheme. This methodology also enables us to evaluate the technical interest rates associated with the different generations involved. As it will be shown, if these reforms are applied without modifications, they will certainly produce intergenerational transfers, and the amount such transfers will increase with the duration of the proposed changes.

The limitations embedded in such redistributions induce us to formulate, from a theoretical point of view, a new type of reforms – i.e. dynamic parametric- to ensure that future generations have identical technical interest rates related to their financial equilibrium.

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[1] Luis Camacho. “Explicitación de las variables que intervienen en el equilibrio financiero individual de un sistema jubilatorio con prestación definida” Banco de Previsión Social. Comentarios de Seguridad Social No. 7 (abril-junio 2005)

[2] Luis Camacho: “Análisis de la tasa de rentabilidad implícita en el equilibrio financiero de un sistema de reparto”. Banco de Previsión Social. Comentarios de la Seguridad Social No 10.

[3] Luis Camacho. “Un modelo heurístico para calcular de la tasa de interés técnico de corte asociada a un sistema de Capitalización Parcial”. Banco de Previsión Social. Comentarios de Seguridad Social No.23 Abril-Junio 2009

[4] Luis Camacho. “La tasa de interés técnico actuarial asociada a un sistema de capitalización completa con prima única”. Banco de Previsión Social. Comentarios de Seguridad Social No.14 .Enero-marzo 2007.

[5] Luis Camacho.”Clasificación de los Sistemas de Financiación Colectiva según el Grado de Capitalización. Banco de Previsión Social. Comentarios de Seguridad Social No. 24. Julio-Setiembre de 2009.

EVALUATION YEAR OF THE FINANCIAL EQUILIBRIUM OF THE SYSTEM

As mentioned above, it is important to project the financial equilibrium equation of the pay as you go scheme in the long term. We will analyze the components of the equation for a specific year –when all members belong to future generations, i.e. as contributors or retirees.

We introduce the following diagram to provide a clear view of the equilibrium equation and the year when it will be evaluated. T=0 represents the beginning of the first year considered in the analysis, which will generally be a close one in the future. In this analysis we considered the year[6] as the time unit; therefore, every period in between two time instances in the graph represents a particular year.

T= 0 1 e_r-e_i e_f-e_i

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|--|--------------|----------------------------------|-
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- We define "ei" as labor force entry age ,"er" as retirement age, and "ef" as the oldest age we can find affiliates alive. T= er-ei is the beginning of the year when members of the generation that joined the labor force in T=0 retire. Therefore, T= ef-ei is the beginning of the year when the oldest retirees would be the members of the mentioned generation.

Let us consider a demographic projection of the system, where entries and exits of the scheme are assumed to be in accordance with dynamic mortality rates during the overall horizon of the study. That projection will allow us to determine the number of entrants per year $\mathbf{A^{(T)}}$, in accordance to the following diagram:

T= 0 1 e_r-e_i e_f-e_i

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|--|--------------|----------------------------------|--
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$A^{(0)}$ $A^{(1)}$ $A^{(er-ei)}$ $A^{(ef-ei)}$

We can now indicate the year when we will evaluate the financial equilibrium of the pay as you go scheme in the future. That year appears last on the time axis (T=ef-ei) as at that point the next contributors' generation, is the oldest one.
In other words, if we assume that in the year that begins in T=0 the next generation starts contributing, the financial equilibrium will refer to the period between **"e_f-e_i"** and **"e_{f+1}-e_i".**

ESTIMATE OF THE AMOUNT OF CONTRIBUTORS AND RETIREES IN THE STUDIED PERIOD

In that year, we can find all contributors and retirees generations which will be associated with new contributors and new retirees between T=0 and T=ef-ei.

Before considering the number of surviving contributors of entrants from all previous years, we will associate the age at that instance with each generation of contributors.
Therefore, the following diagram displays the mentioned ages as sub indexes per year.

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[6] The time unit should be the month as both contributions and pensions are collected and paid monthly. In order to carry out the analysis with this time unit, it is necessary to count with mortality tables for monthly periods.

T=	0	1	er-ei	ef-ei
	|-----|	-------------|	---------------------------------|	--
	$A_{ef}^{(0)}$	$A_{(ef-1)}^{(1)}$	$A_{(ef-er+ei)}^{(er-ei)}$	$A_{ei}^{(ef-ei)}$
	$l_{ef}^{(0)}$	$l_{(ef-1)}^{(1)}$	$l_{(ef-er+ei)}^{(er-ei)}$	$l_{ei}^{(ef-ei)}$

On the last row we place the expected number of survivors from the cohorts associated with the different labor force entry years. In every case we assume $\mathbf{l_{ei}}$ as the initial figure. In this case, we use dynamic mortality tables as sources of information.

Therefore, $\mathbf{lj^{(ef-j)}}$ is equal to the expected number of survivors at age j from the cohort that joins the labor market in **T=ef-j** year with **(ei <=j <=ef)**.

We can now formulate the expression to estimate the number of survivors for the last year considered with the following rule of three:

$$\mathbf{l_{ei}} \quad --- \quad \mathbf{l_j^{(ef-j)}}$$

$$\mathbf{A_j^{(ef-j)}} \quad --- \quad \textbf{Surviving affiliates of age j}$$

The number **Affiliates of "J" years in ef-ei** will be denoted as follows:

$$\mathbf{ASj = A_j^{(ef-j)} * l_j^{(ef-j)} / l_{ei}} \qquad \textbf{[1]}$$

Considering that the quotient $\mathbf{lj^{(ef-j)}/l_{ei}}$ is equal to the probability of being alive at age j in T=ef-ei for a person who initiates its activity in T=ef-j.

When using the term Affiliates, we refer to both contributors and retirees. Therefore, age **(j)** will be the variable that will enable us to know if the affiliate is a contributor **($j<e_r$)** or a retiree **($j>=e_r$).**

The financial equilibrium analysis of the pay as you go scheme in such a distant horizon has two implications:

1) The objective is to evaluate the associated financial effect on future generations of affiliates, not considering the current features of the scheme. Therefore, this type of analysis could go in hand with financial-actuarial projections, as it only displays the distant future, not considering financial problems in between. However, we considered this study relevant to evaluate possible parametric changes as they could have an impact on new generations of affiliates.

2) Although this type of analysis supplements the financial projections, it requires them to have a very long time horizon as they provide information on the evolution of the number of entrants (future contributors). It is important to point out that if the initial age for joining the labor force is 20 and the mortality table sets 100 years as maximum age, it will be necessary to have information on the number of individuals joining the labor market for approximately an 80 year period.

LONG TERM GENERIC EQUILIBRIUM EQUATION OF THE SCHEME

By definition, the financial equilibrium of a pay as you go scheme, is defined each year through the equality between contributions revenues and benefits payments. In this analysis we introduce specific formulations that allow to estimate the number of contributors and retirees for a given year. The introduction of financial variables enable us to set the level of contributions and pensions for that year.

Contributors and Retirees

We can express the amount of contributors and retirees by the relation [1] as we know the age at which individuals join the labor market (ei) and the retirement age (er). We have to take into account that in the initial year "T=ef-ei", those individuals younger than age "er" will be contributors and the rest will be retirees.

Therefore, the total number of contributors and retirees results from the accumulation of the affiliates in two categories according to age:

$$\textbf{Contributors} = \sum_{j=e_i}^{j=e_{r-1}} [\, ASj \,] \qquad [2]$$

$$\textbf{Retirees} = \sum_{j=e_r}^{j=e_f} [\, ASj \,] \qquad [3]$$

The first summation expresses all contributors older than **"e_i"** and younger than "$\mathbf{e_{r1}}$"(retirement age). We assume that the old age pension is paid since age **"e_r"** .

The second summation expresses the total number of retirees per year, therefore the upper bound indicates the number of retirees who reach the oldest possible age in accordance to the amplitude of the mortality table taken into account.

Contributions and Pensions

We assume that the initial salary at age "e_i" increases annually as a result of improvements in the labor career; therefore, we can set different salaries taking into account age.

$$\textbf{SALARY at AGE } j = S_j$$
$$\textbf{given } j => e_i$$

It is important to point out that in this analysis we only consider salary increases by vertical salary mobility; even though salaries increase as a result of productivity growth. Monetary values considered in this study are deflated by the general salary index. If we assume a zero global salary increase (as it is our case), results would be the same.

On the other hand, we assume that the new retirees' pensions are calculated by multiplying the retirement basic salary (SBJ) by the replacement rate (TR)

$$\textbf{RETIREMENT at AGE J} = JRj = SBJ * TR$$
$$\textbf{given } j >= er$$

We assume that pensions are indexed to salaries[7]; therefore, as in the above relation, values are also deflated by the general salary index.

Total contributions and pensions of the studied year could be expressed as follows:

$$\textbf{Contributions} = \sum_{j=e_i}^{j=e_{r-1}} [Sj^* \, ASj] * TCR = SMC^{(D)} * \sum_{j=e_i}^{j=e_{r-1}} [ASj] * TCR \qquad [3]$$

Where **SMC(D)** is the contribution average salary[8] and **TCR** is the contribution rate in a pay as you go scheme. The latter is the adjustment variable in a defined benefit scheme.

$$\textbf{Pensions} = SBJ*TR * \sum_{j=e_r}^{j=e_f} [ASj] \qquad [4]$$

Now, we can formulate the equilibrium equation of the pay as you go scheme, which can be verified in the following simple way:

Contributions [3] = Pensions [4]

$$SMC^{(D)} * \sum_{j=e_i}^{j=e_{r-1}} [ASj] * TCR = SBJ*TR * \sum_{j=e_r}^{j=ef} [ASj]$$

In order to make this equality possible, we should assume a system financed exclusively by affiliate contributions, with irrelevant management expenses (otherwise they should be considered in the equilibrium equation).

SPECIFIC FORMULATION OF THE FINANCIAL EQUILIBRIUM EQUATION

We will reformulate the financial equilibrium equation including the following variables: contribution and retirement average periods, and new entrants associated with contribution and retirement central ages.

Contribution and Retirement Average Periods

Let us assume a hypothetical cohort with the following development

$$\{ l_{ei}^{(ef-ei)}; l_{ei+1}^{(ef-ei-1)}; l_{ei+2}^{(ef-ei-2)}; \ldots\ldots\ldots; l_{ei+h}^{(ef-ei-h)}; \ldots\ldots\ldots l_{ef}^{(ef-ef)} \}$$

Given $l_{ei}^{(ef-ei)} = l_{ei}$ (as all the cohorts considered in this study start at "l_{ei}")

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7 Other situations are analizad in Luis Camacho: "La incidencia de la fórmula de cálculo del Sueldo Medio Básico Jubilatorio en el equilibrio financiero individual". Banco de Previsión Social. Comentarios de Seguridad Social No. 11 (abril-junio 2006).

8 SMC(D) =Σ [Sj* ASj]/ Σ [ASj], the summations are true for working active ages "ei" and "er-1".

At age **j** the number of survivors can be expressed as $\mathbf{l_j^{(ef\text{-}j)}}$ (these factors are also present in contributions and pensions expressions).

Example:

The following table shows the survival probabilities for the different cohorts, T and J variables are expressed in decades.

CUADRO 1

PROBABILIDADES DE SOBREVIVENCIA POR EDAD

EDAD J	Instante T 0	1	2	3	4	5	6	7	8
2	**1**	1	1	1	1	1	1	**1**	1
3		**0.9910**	0.9938	0.9957	0.9970	0.9974	0.9977	**0.9981**	0.9984
4			**0.9804**	0.9851	0.9886	0.9903	0.9907	**0.9910**	0.9914
5				**0.9597**	0.9678	0.9721	0.9738	**0.9741**	0.9745
6					**0.9177**	0.9278	0.9320	**0.9336**	0.9353
7						**0.8254**	0.8346	**0.8383**	0.8420
8							**0.6456**	**0.6528**	0.6601
9								**0.3210**	0.3246

J Y T EXPRESADOS EN DÉCADAS

Survival probabilities can be visualized diagonally, the first cohorts are shown along the lower diagonal and the cohorts that join the labor market in later dates are shown in upper levels.

Given T=ef-ei equal to T=7, the column highlighted in bold represents survival probabilities for the hypothetical cohort defined above.

We can define the contribution average period (**TMC$^{(D)}$**) as the sum of survival probabilities of that hypothetical cohort for every contribution age.

$$\mathbf{TMC^{(D)}} = \sum_{j=e_i}^{j=e_{r-1}} \left(l^{(ef-j)}_j / l_{ei}\right) \qquad [5]$$

Each quotient of the summation indicates the fraction of the year that the contributor is expected to live since he/she starts working. As these fractions accumulate for the whole working period, the result will be equal to the expected contribution period. We assume that deaths are verified at the end of every year.

According to Table 1, if we consider the labor market entry age as j=2 and the retirement age as j=7, we can calculate the contribution average period by adding the first five values of the column in bold characters. The result is TMC= 4,9 or a 49 year contribution period.

The sum of survival probabilities for all retirement ages can be expressed as **the retirement average period** (TMJ $^{(D)}$).

$$\mathbf{TMJ^{(D)}} = \sum_{j=er}^{j=e_f} \left(l^{(ef-j)}_j / l_{ei}\right) \qquad [6]$$

Taking into account table 1, we can calculate this average period, adding the last three figures of the column highlighted in bold. The resultant retirement average period is 1.81, or 18.1 years.

This last result must be considered carefully, as it represents the expected average time units with retirement benefits, viewed from labor force entry date (first contribution).

Contribution and Retirement Central Ages associated with entrants and new retirees.

Let us assume that the scheme has a constant level of entrants per year, equivalent to that of an intermediate year "ef-ECC", and that the level of expected contributors for the considered year is similar to that expressed in **[2].**

We should define the constant number of entrants as $AECC^{(ef-ECC)}$ taking into account **[1]** for the following relation to be true:

$$\textbf{Contributors} = \sum_{j=e_i}^{j=e_{r-1}} [ASj] = A_{ECC}^{(ef-ECC)} * \sum_{j=e_i}^{j=e_{r-1}} l_j^{(ef-j)} / l_{ei}$$

Considering **[5]**, the new expression for the number of contributors can be:

$$\textbf{Contributors} = A_{ECC}^{(ef-ECC)} * TMC^{(D)} \qquad [7]$$

"ECC" is contribution central age.

Therefore, the number of contributors for the year beginning with T=ef-ei, is equal to the number of new contributors associated with a contribution central age multiplied by the contribution average period.

It is important to point out that the number of entrants considered would exhibit an intermediate level regarding the level of entrants for current contributors.

$$Aei^{(ef-ei)}; Aei+1^{(ef-ei-1)}; \ldots\ldots; Aer-2^{(ef-er+2)}; Aer-1^{(ef-er+1)}$$

Subscripts indicate the age in the last year (of corresponding contributors), ranging from ei to er-1, and as we consider the constant number of corresponding entrants intermediate, ECC is also between contribution limit ages, i.e. contribution central age.

We can calculate the number of total retirees as follows,

$$\textbf{Retirees} = A_{ECJ}^{(ef-ECJ)} * TMJ^{(D)} \qquad [8]$$

"ECJ" = retirement central age.

If we replace the annual number of entrants corresponding to the retirement age in the studied year, by a constant level of entrants corresponding to an intermediate year "ef-ECJ"", the total number of retirees will be similar to that expressed in **[3].**

Contributions and Pensions in accordance with entrants and new retirees at central ages

Taking into account the expression mentioned above, the equilibrium equation of the scheme for the considered year can be formulated as follows:

$$\textbf{Contributors} = A_{ECC}^{(ef-ECC)} * TMC^{(D)} * SMC^{(D)} * TCR \qquad [9]$$

The first two factors indicate the total number of contributors in the last year, the third one refers to contribution average salary, and the last one represents the contribution rate, which is the adjustment variable.

It is important to highlight that the factors that explain the contribution level are not independent factors, except the contribution rate.
On the other hand, the contribution average salary will be influenced by the variables that have an impact on the contribution central age, and the contribution average period will be influenced by those variables that explain the contribution average salary. Any change in any of these factors can cause changes in the value of the other ones. For example, an adjustment of survival probabilities will cause adjustments on ECC, SMC and TMC.

$$\textbf{Pensions} = A_{ECJ}^{(ef-ECJ)} * SBJ * TR * TMJ^{(D)} \qquad [10]$$

The new formulation of total pensions depends on the retirement basic salary, the number of entrants associated with retirees whose age in the last year is the retirement central age, the replacement rate and the retirement average period.

Equilibrium Equation and Entrants at Central Ages

As we have already mentioned the financial equilibrium for the pay as you go scheme is analyzed for the year that begins in T= “ef-ei”. Therefore, it is important to express the equality between contributions and benefits for that year. That equality will be achieved by setting the contribution rate of the scheme **(TCR).**

Therefore, the equilibrium between **[9]** contributions and **[10]** pensions is obtained, when **TCR** satisfies the following equality:

$$TCR = \frac{[\ SBJ * TR]\ /\ [\ SMC^{(D)}]}{[TMC^{(D)} / TMJ^{(D)}] * [\ A_{ECC}^{(ef-ECC)}\ /\ \ A_{ECJ}^{(ef-ECJ)}\ ]} \qquad [11]$$

The numerator expresses the relation between the average pension and the contribution average salary. The denominator expresses the product of two quotients and the result shows the relation between the average number of contributors and retirees. Therefore, we can express the basic relations that influence the equilibrium rate as follows:

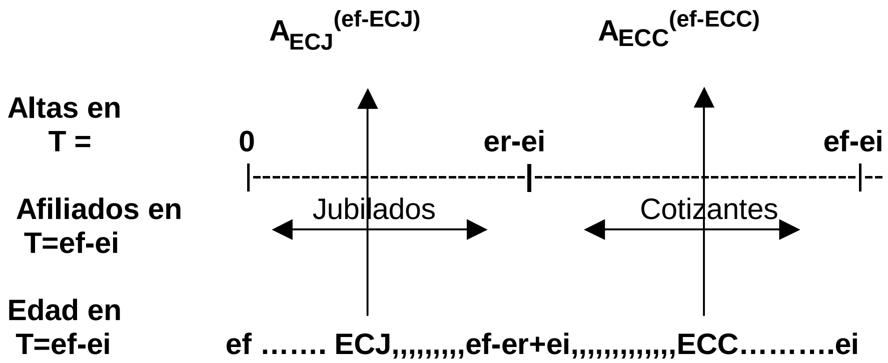
$$TCR = \frac{\text{economic relation}}{\text{demographic relation}}$$

The equilibrium contribution rate of the pay as you go scheme varies directly proportional to the economic relation variation and inversely proportional to the demographic relation.

The demographic relation can be disaggregated into several components, enabling us to estimate the level of technical interest associated with the pay as you go scheme in an equilibrium condition.

EQUILIBRIUM CONTRIBUTION RATE OF THE SCHEME AND TECHNICAL INTEREST RATE ASSOCIATED WITH THE PAY AS YOU GO SCHEME

We will locate new entrants (contributors) associated with contribution and retirement central ages on the time axis:



We can see that entrants associated with contribution central age **($A_{ECC}^{(ef-ECC)}$)** are verified after **$A_{ECJ}^{(ef-ECJ)}$.** Therefore, if we assume that the scheme is under expansion the number of the later entrants will be larger.

Even if the above property is not verified, it is important to formulate the relation between both entry ages which lead to the following expression:

$$A_{ECC}^{(ef-ECC)} / A_{ECJ}^{(ef-ECJ)} = 1 + c(ef\text{-}ECJ, ef\text{-}ECC) \qquad [12]$$

Where **c(ef-ECJ,ef-ECC)** is the entrants growth rate between **"ef-ECJ"** and **"ef-ECC"**. If the system is under expansion, the result will be positive.

As the length of the period (with both types of entrants) is equal to the difference between retirement and contribution central ages **(ECJ-ECC)**, we can define a constant rate **"$c^{(D)}$"** to fulfill the following relation:

$$(1 + c^{(D)})^{(ECJ-ECC)} = 1 + c(ef\text{-}ECj, ef\text{-}ECC) \qquad [13]$$

Therefore, **"$c^{(D)}$"** would be the contributors annual average growth for a specific period **(ef- ECJ, ef- ECC)** -of length ECJ-ECC- or recovery period.

The expression **[11]** can be reformulated as follows:

$$TCR = \frac{TMJ^{(D)}}{TMC^{(D)}} * \frac{SBJ\ .TR}{SMC^{(D)}} * (1 + c^{(D)})^{ECC-ECJ} \qquad [14]$$

We arrive at a similar formulation in a previous analysis, where we assume mortality rates which do no change along time[9] . In that study we demonstrate that **"c"** can be

[9] Luis Camacho: "Análisis de la tasa de rentabilidad implícita en el equilibrio financiero de un sistema de reparto". Banco de Previsión Social. Comentarios de la Seguridad Social No 10. Enero-Marzo 2006

associated with the technical interest rate of the pay as you go scheme for the year that begins in **T= ef-ei.**
An additional result is also shown here, as **"c" rate** is equivalent to the technical interest rate associated with those members of the cohort who enter the labor market in the first year of the analysis horizon (T=0).

Therefore, we can conclude that the technical interest rate of a pay as you go scheme **"i_R"** for the year that begins in **T=ef-ei** would be equal to **"$c^{(D)}$"**.

Regarding the second property analyzed above, **"$c^{(D)}$"** will not be the same as the technical interest rates associated with any future generations of the pay as you go if mortality is dynamic.

Example:

In the following example we calculate the long term technical interest rate of a pay as you go scheme.

Assumptions:

- *Survival probabilities as shown in Table 1*
- *Labor market entry age : j=2 and*
- *Retirement age :j=7.*
- *Constant contribution salary along the whole horizon of the study: 10.000 monetary units*
- *Basic retirement salary : similar level*
- *Replacement rate: 60%*

Values are expressed in terms of constant salaries, assuming no salary vertical mobility. It is necessary that the demographic projections provide information on actual new contributors. In our example this information appears in columns 2 and 3.

Table 2 also shows survival probabilities, (which are equal to those in the last column of Table 1), contributors by age at the decade starting in T=7, salary , total salaries by age in the that decade. The last column shows the product of the salary (column V) multiplied by the survival probabilities (column III).

CUADRO 2

MASA SALARIAL

DECADA QUE SE INICIA EN T=7

EDAD EN T=7	ALTAS T=	ALTAS CANTIDAD	PROBABILIDAD SOBREVEVIENCIA	COTIZANTES ACTUALES	SUELDO UNITARIO	MASA SALARIAL	
J	I	II	III	IV	V	VI	III*V
2	7	14,053	1	14053	10000	140,531,420	10,000
3	6	13,778	0.998055	13751	10000	137,507,869	9,981
4	5	13,376	0.991036	13256	10000	132,564,007	9,910
5	4	12,862	0.974139	12529	10000	125,292,090	9,741
6	3	12,249	0.933623	11436	10000	114,362,833	9,336
TOTALES			**4.897**			**650,258,218**	**48,969**

J Y T EXPRESADOS EN DÉCADAS

Taking into account the figures shown on the last row we are able to find the most relevant values:

$TMC^{(D)} = 4.897$ and $A_{ECC}^{(ef\text{-}ECC)} = 650,258,218 / 48,969 = 13,279$

Therefore, the contribution average period is 4.897 decades and the entrants associated with those who show the contribution central age at T=7 are 13,279 units. This value allows us to calculate the contribution central age. According to Table 2, the ages (j) in T=7 which show the closest figure of associated entrants with 13,279 are J=4 and J=5. We can determine the contribution central age through linear interpolation between the number of entrants at those ages. In this case ECC= 4,189 or 41.89 years.

Expression [12] will enable us to calculate the level of contributions for the last year taken into account

*Contributions = 4,897 * 10,000 * 13,279* TCR*

The following table provides information on pensions:

Table 3

IMPORTES DE JUBILACIONES

EDAD EN T=7	ALTAS T=	ALTAS CANTIDAD	PROBABILIDAD SOBREVEVIENCIA	JUBILADOS ACTUALES	JUBILACION UNITARIA	JUBILACIONES TOTALES	
J	I	II	III	IV	V	VI	III*V
7	2	11,556	0.838306	9687	6000	58,124,763	5,030
8	1	10,800	0.652819	7050	6000	42,302,689	3,917
9	0	10,000	0.321041	3210	6000	19,262,466	1,926
TOTALES			**1.812**			**119,689,918**	**10,873**

J Y T EXPRESADOS EN DÉCADAS

Taking into account the figures shown on the last row (totals) we are able to find the most relevant values

$TMJ^{(D)} = 1.812$ and $A_{ECJ}^{(ef\text{-}ECJ)} = 119{,}689{,}918 / 10{,}873 = 11{,}008$

In column II, the ages in T=7, which show the closest number of entrants associated with ECJ, are J=7 and J=8. Through linear interpolation between those figures we can determine the retirement central age- ECJ= 7,725 or 77,3 years.

In this case, the specific values for the expression [13] would be:

$$Pensions = 1{,}812 * 10{,}000 * 0.6 * 11{,}008 = 119{,}689{,}918$$

We can now calculate the level of the equilibrium contribution rate [14]:

$$TCR = \frac{1.812}{4.897} * \frac{10.000*0.6}{10.000} * \frac{11.008}{13.279} = 18.41\%$$

Therefore, the equilibrium contribution rate is 18,41% of total salaries (masa salarial).
However, it is also relevant to calculate the technical interest rate associated with the pay as you go scheme for the period starting at T=7, consequently we use the expression [13], to find "$c^{(D)}$".

$$(1 + c^{(D)})^{(7{,}725-4{,}189)} = 13{,}279/11{,}008 = 1.2063$$

We arrive at $c^{(D)} = 5{,}45\%$, which represents the ten-year technical interest rate of the pay as you go scheme, and which is equivalent to a 0,53% actual annual rate

PARAMETRIC ADJUSTMENTS OF THE PAY AS YOU GO SCHEME

It is important to point out, again, that neither current financial condition of the pay as you scheme nor the medium term results are relevant for this study. This paper evaluates long term changes that will affect future generations in a very long term horizon.

Therefore, this study will enable us to show that the financial aspects of the pay as you go scheme are based on the strategic changes that will have impact in the far future, regardless of the historical evolution of the scheme.

Likewise, the above mentioned results will be useful to carry out parametric changes in a simple way, considering the overall equilibrium of the pay as you go scheme.

Such changes can be divided into two categories; those that address contribution rates, replacement rates and retirement basic salaries, and those that address retirement ages. These categories could be combined, but we study them separately to simplify the analysis.

1) Contribution rate, replacement and basic retirement salary changes

The Retirement Basic Salary (SBJ) is calculated as the average of working life salaries used to determine the initial pension. The number of working years considered, and the index employed to update salaries at the moment of retirement will also impact on the SBJ calculation.
Regarding retirement basic salaries, changes can be less relevant than contribution and replacement rate changes. However, there are certain instances where changes in the retirement basic salary become important, especially when it is calculated as the average of the last working years and/or without any indexation.

In any case, the relation among these three parameters could be formulated as follows (taking the equilibrium equation expressed in **[14]** as the departure point):

$$TCR = K * SBJ * TR \qquad [15]$$

$$\text{given} \quad K = \frac{TMJ^{(D)} * (1 + c^{(D)})^{ECC-ECJ}}{TMC^{(D)} * SMC^{(D)}}$$

Any combination of **TCR, SBJ and TR,** that fulfills the equality **[15]** should be considered as an alternative for a possible parametric reform.

Consequently, there exists a wide scope for changes, i.e. flexible level, as it is possible to reform some or all the parameters keeping the long term financial equilibrium.

Example:

We can analyze several possibilities regarding changes on TCR and TR. However, we can not formulate changes on the retirement basic salary, as we assume no vertical salary mobility.

Nevertheless, we can appreciate that in this case, the following relation between TCR and TR is true:

$$0.306842 * TR = TCR$$

Therefore, multiple combinations of TR and TCR that fulfill the previous relation are valid. The following table displays some possible options:

	CASO 1	CASO 2	CASO 3	CASO 4	CASO 5	CASO 6
TCR	**20,00%**	**19,00%**	**18,00%**	**17,00%**	**16,00%**	**15,00%**
TR	**65,18%**	**61,92%**	**58,66%**	**55,40%**	**52,14%**	**48,89%**

In spite of the fact that this study does not consider the present situation together with the reform for future generations, it is advisable to evaluate the transition changes so as to achieve continuity in the scheme evolution. Consequently, even though current entitlements (depending on seniority) are already acquired rights, it will be necessary to modify them partially, by applying gradually the projected reforms to future generations.

Let us assume that today the financial equilibrium is achieved at a 18% contribution rate (which can not be increased due to the heavy tax burden) and at a 70% replacement rate. As there exists a significant gap between this current replacement rate and the future replacement rate for new generations, consecutive decreases of the level of this variable should be carried out for the current generations so as to reach the projected 60 % in a gradual way.

2) Changes in retirement age

In order to evaluate effectively any increase in retirement ages, it is necessary to carry out new demographic projections of the scheme, taking into account these changes. This way, we would have information on the new evolution of the expected number of entrants during the overall horizon of the analysis.

With that information available, we can evaluate the new contribution and replacement rates. For that purpose we can use the expressions associated with the long term financial equilibrium equation of the scheme.

THE INDIVIDUAL EQUILIBRIUM EQUATION OF NEXT COHORT MEMBERS

We assume that the next cohort joins the labor market at T=0.
Therefore, the evolution could be:

$$\{ l_{ei}^{(0)}; l_{ei+1}^{(0)}; l_{ei+2}^{(0)}; \ldots\ldots\ldots; l_{ei+h}^{(0)}; \ldots\ldots\ldots l_{ef}^{(0)} \}$$

Given $l_{ei}^{(0)} = l_{ei}$ as every cohort considered for this study starts at **"l_{ei}" level.**

Furthermore[10], in accordance with a previous analysis, it is possible to express the individual equilibrium equation through the equality between the present values of contributions and the expected retirement pensions as projected for a typical member of that cohort. In the mentioned analysis, we assume mortality rates constant, which can also be the case here, keeping in mind the special evolution of the cohort mentioned above.

In this case, it is important to highlight the way central ages are calculated regarding contribution central age **ECC (0)**, and where the following equation for contribution present value at **T=0** is true**:**

$$VAC^{(0)} = [\sum_{j=e_i}^{j=e_{r-1}} Sj * l^{(0)}_j \ / \ l^{(0)}_{ei}] * TC * (1+i)^{(ei-ECC(0))} \qquad [16]$$

Where **"TC"** is the contribution equilibrium rate and **"i"** is the technical interest rate associated with the cohort in the financial equilibrium.

Pensions present value can be expressed as :

$$VAJ^{(0)} = [\sum_{j=e_r}^{j=e_f} SBJ*TR * l^{(0)}_j \ / \ l^{(0)}_{ei}] * (1+i)^{(ei-ECJ(0))} \qquad [17]$$

We can also express the equivalence of the present values as follows:

[10] Luis Camacho. "Explicitación de las variables que intervienen en el equilibrio financiero individual de un sistema jubilatorio con prestación definida" Banco de Previsión Social. Comentarios de Seguridad Social No. 7 (abril-junio 2005)

$$TMC^{(0)} * SMC * TC * (1+i)^{(ei-ECC(0))} = TMJ^{(0)} * SBJ * TR * (1+i)^{(ei-ECJ(0))} \qquad [18]$$

Superscripts **"0"** indicate that in this case, calculations are carried out based on **"$l^{(0)}_j$"** for the cohort that starts working at **T=0**.

The first side of the equation represents the present value of individual contributions and the second one refers to benefits.
$TMC^{(0)}$ and **$TMJ^{(0)}$** represent contribution and retirement average periods when dynamic mortality rates are considered in the evolution of a cohort; **SMC and SBJ** are contribution average salary and retirement average basic salary; **TC and TR** are contribution and replacement rates; and **ECC (0)** and **ECJ (0)** are contribution and retirement central ages.

Let us assume that every member of this cohort contributes to the scheme at the pay as you go rate **(TCR)** defined by the expression **[14]**, and that **TC=TCR** is true. If we also assume that the replacement rate is the same that the pay as you go one, the interest rate "i" will be the adjustment variable of the equation **[19]** instead of **TC**. Likewise, this variable will also influence **$ECC^{(0)}$** and **$ECJ^{(0)}$** .

Having defined TC=TCR, TR and SBJ, we still have to find one variable, i.e. the associated technical interest rate, **($i^{(0)}$),** to achieve the financial equilibrium between the present value of income and expenditure.

Thus, the equality **[19]** can be reformulated as follows:

$$(1+i^{(0)})^{-ECC(0)+ECJ(0)} = \frac{TMJ^{(0)} * SBJ * TR}{TMC^{(0)} * SMC * TCR} \qquad [19]$$

Therefore, it is enough to apply the formula mentioned above to find the technical interest rate, which in this case is equivalent to the expected rate of return that members of the first cohort would receive in the pay as you go scheme with predefined **TCR, TR and SBJ**.

It is also important to take into account that a previous analysis on the pay as you go scheme[11] demonstrates that if mortality rates do not vary along time, the property stating the similarity between **$i^{(0)}$** and **i_r**[12]**,** will be true**.** Therefore, the technical interest rate associated with the pay as you go scheme will also be equivalent to the technical interest rate associated with a contributor who starts working at T=0.

If we introduce dynamic mortality rates, that property may not be true, because the survival probabilities considered in the financial equilibrium equation of the pay as you go scheme are not the same as the ones for the initial cohort.

[11] Luis Camacho: "Análisis de la tasa de rentabilidad implícita en el equilibrio financiero de un sistema de reparto". Banco de Previsión Social. Comentarios de la Seguridad Social No 10.

[12] As entrants growth rate could vary in the horizon of the study, the equality between the technical interest rate of the initial generation and that of the pay as you may not be verified. In addition to this, the rate of the pay as you go scheme is not calculated for the whole horizon, but for the period ECJ-ECC (amplitude recovery period). Nevertheless, the difference between both rates is not relevant.

Let us analyze the following example:

Example:

Considering expression [19], survival probabilities for this initial cohort are shown in Table 1, (lower diagonal). We can calculate, by successive iterations, the specific level of interest rate.

Entrants at T=0

$$(1.05035)^{(-3.911+7.705)} = \frac{1.792 * 10.000 * 0.6}{4.849 * 10.000 * 0.1841}$$

Therefore, a member of the initial cohort with a 18,41% contribution rate and a 60%, replacement rate, will have a 5,03% technical interest rate (return rate) in the decade, equivalent to a 0,49% annual rate. If we compare this result with the 0,53% technical interest rate of the pay as go scheme shown in the example, the difference will not be significant.

We will calculate the technical interest rate associated with the following generation. For this purpose we will use the survival probabilities shown on the diagonal associated with those contributors who start working at T=1. In this case the following equivalence expressed in [19] is true :

Entrants at T=1

$$(1.0533)^{(-3.914+7.705)} = \frac{1.812 * 10.000 * 0.6}{4.875 * 10.000 * 0.1841}$$

This new generation will have a 5,33% ten year technical interest or a 0,52% annual one. Consequently, the two successive generations will have different interest rates.
The later generation will have an increasing rate. It can be demonstrated that this property is true for the following generations, therefore the interest rates increase in the long term.

If we differentiate the rates of return associated with each generation by the superscript that indicates the year of beginning of the activity, the following relation will be true

$$i^{(0)} <= i^{(1)} <=; i^{(2)} <=............<= i^{(ef-ei)}$$

As we assume permanent improvements on mortality rates, the different generations should have increasing contribution rates associated to obtain fix replacement rates. Due to the fact that we assume constant contribution rates, the adjustment variable is the rate of return, which as a result of the improvements in mortality, should be higher along time.

DYNAMIC PARAMETRIC ADJUSTMENTS

The most significant result of the above analysis indicates that a parametric reform with single contribution and replacement rates, constant along time, will necessarily imply that the different generations involved have associated increasing rates of return, also different from the technical interest rates of the pay as you go system in the long term.
As we assume the same salary mobility and retirement basic salaries for all generations, the differences are the result of the permanent changes considered on survival probabilities (as a direct consequence of the mortality rates improvement).

According to the discussed reform, variable contribution and replacement rates along time and according to age may compensate the differences in probabilities. The main purpose of setting these rates is to provide all future generations with the same rate of return. Likewise, the technical interest rates associated with the pay as you will also have the same level.

How can we achieve this objective? In the financial equilibrium for the initial generation we can find the fixed contribution **(TCR)**, replacement **(TR)** and technical interest **($i^{(0)}$)** rates. We formulate variable expressions for the future contribution and replacement rates, so as $i^{(0)} = i^{(0)} = i^{(1)} = i^{(2)} = i^{(3)} = ... = i^{(ef-ei)}$ to be true. It is possible to demonstrated that this property is true in the following cases:

i) **The contribution rate by age for a generation that initiates its activity in year "t" at age "j" is:**

$$TC_j^{(t)} = [\, I_j^{(0)} / I_j^{(t)} \,] * TC \quad (ei<=j<=er-1) \qquad [20]$$

ii) **The replacement rate by age for a generation that initiates its activity in year "t" at age "j" is:**

$$TR_j^{(t)} = [\, I_j^{(0)} / I_j^{(t)} \,] * TR \quad (er<=j) \qquad [21]$$

We will demonstrate that the technical interest rate (return) does not vary by addressing different possible cases.

1) **Different generations**. Let us consider a generic generation that starts working at "t".

By adapting expressions [16] and [17], we can formulate contribution and pension present values associated with this cohort as follows:

$$VAC^{(t)} = [\sum_{j=e_i}^{j=e_{r-1}} Sj * \frac{Ij^{(t)}}{Iei} * TCj^{(t)}] * (1+i)^{(ei-ECC)} \qquad [22]$$

$$VAJ^{(t)} = [\sum_{j=e_r}^{j=e_f} SBJ*TR\, j^{(t)} * \frac{Ij^{(t)}}{Iei}] * (1+i)^{(ei-ECJ)} \qquad [23]$$

If we replace $TCj^{(t)}$ and $TRj^{(t)}$ by the right hand side of expressions [20] and [21], the previous present values, the resulting expressions will be the same as the ones for the initial generation; therefore, the financial equilibrium equation will be formulated as follows:

$$VAC^{(t)} = VAC^{(0)}$$
$$VAJ^{(t)} = VAJ^{(0)}$$

As values are exactly the same as those for the first generations, the technical interest rates are also the same in the financial equilibrium, then we can conclude that $i^{(t)} = i^{(0)}$ is true.

2) Pay as you go scheme system. We formulate the following general expressions for contributions and pensions for the year ef-ei, based on the results of **[1], [3] and [4].**

j=er-1

$$\text{contributions} = SMC^{(D)} * \sum_{j=e_i} \left[A_j^{(ef-j)} * \frac{l_j}{l_{ei}}^{(ef-j)} * TCR_j^{(ef-j)}\right]$$

$$\text{pensions} = SBJ * \sum_{j=e_r}^{j=e_f} \left[A_j^{(ef-j)} * \frac{l_j}{l_{ei}}^{(ef-j)} * TR_j^{(ef-j)}\right]$$

If we replace **TCR $j^{(t)}$** and **$TRj^{(t)}$** by the right hand side of **[20] and [21]**, the survival probabilities, with **TCR and TR** constant, are those of the first generation. Therefore, the financial equilibrium equation for the year that begins in “ef-ei” will be:

$$TCR = \frac{TMJ^{(0)}}{TMC^{(0)}} * \frac{SBJ\ .TR}{SMC^{(0)}} * (1+ i_R)^{ECC-ECJ}$$

All values are exactly the same as those of the **financial equilibrium equation** for the first generation; therefore, in the financial equilibrium, the technical interest rates of the pay as you go scheme also coincide with the ones for that cohort: $i_R = i^{(0)}$.

Example:

We consider the case mentioned before where survival probabilities are shown in Table 1.

The financial equilibrium equation for the different generations, shows the following significant values: TC = 18,41% and TR = 60%.

Consequently, according with [21] and [22] the following is true:

$$TCj^{(t)} = [\, lj^{(0)} // lj^{(t)} \,] * 0.1841 \qquad j<7$$

$$TRj^{(t)} = [\, lj^{(0)} // lj^{(t)} \,] * 0.6 \qquad j>=7$$

Taking into account Table 1, we can introduce a new table where we include the coefficients which multiply the contribution and replacement basic rates in the right hand side of the previous expression.

CUADRO 4
COEFICIENTES APLICABLES A LAS TASAS FIJAS

	Instante T							
EDAD J	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3		1.0000	0.9972	0.9953	0.9940	0.9937	0.9933	0.9930
4			1.0000	0.9952	0.9917	0.9899	0.9896	0.9892
5				1.0000	0.9916	0.9872	0.9855	0.9851
6					1.0000	0.9890	0.9846	0.9829
7						1.0000	0.9890	0.9846
8							1.0000	0.9890
9								1.0000

J Y T EXPRESADOS EN DÉCADAS

Therefore, it is possible to obtain the effective contribution or replacement rates for each time unit and age, by multiplying the coefficients (Table 4) by the contribution or replacement basic rates. The results for the last period are shown below

CUADRO 5
TASAS APLICABLES PARA T ENTRE 7 Y 8

EDAD J	coef.	tasa basica	tasa variable
2	1.0000	0.1841	**0.1841**
3	0.9930	0.1841	**0.1828**
4	0.9892	0.1841	**0.1821**
5	0.9851	0.1841	**0.1813**
6	0.9829	0.1841	**0.1809**
7	0.9846	0.6	**0.5908**
8	0.9890	0.6	**0.5934**
9	1.0000	0.6	**0.6000**

Contribution rates for ages below 7 are shown in the last column, and effective replacement rates are shown for the other ages.

Contribution and replacement rate levels are shown for the last period studied which starts at T=7. The levels are below those of the initial generation, except for the initial and final ages

Additionally, if we consider that $(Ij^{(t)}/ I_{ei})* TCj^{(t)}$ *and* $(Ij^{(t)}/ I_{ei})* TRj^{(t)}$ *appear in [23] and [24] and that [21] and [22] formulate the expressions for* $TCj^{(t)}$ *and* $TRj^{(t)}$ *, then the final results are:*

$$(Ij^{(0)}/ I_{ei})* TC \quad \text{and} \quad (Ij^{(0)}/ I_{ei})* TR$$

From a financial point of view, the individual financial equilibrium of a member of any generation is also obtained considering single contribution and replacement rates equal to the basic rates, and survival probabilities equal to those of the first generation.

In order to analyze the financial equilibrium, we can consider the probabilities shown in the following table.

CUADRO 6
PROBABILIDADES DE SOBREVIVENCIA APLICABLES CON TASAS FIJAS

	Instante T							
EDAD J	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3		**0.9910**	**0.9910**	**0.9910**	**0.9910**	**0.9910**	**0.9910**	**0.9910**
4			**0.9804**	**0.9804**	**0.9804**	**0.9804**	**0.9804**	**0.9804**
5				**0.9597**	**0.9597**	**0.9597**	**0.9597**	**0.9597**
6					**0.9177**	**0.9177**	**0.9177**	**0.9177**
7						**0.8254**	**0.8254**	**0.8254**
8							**0.6456**	**0.6456**
9								**0.3210**

J Y T EXPRESADOS EN DÉCADAS

In Table 6, probabilities appear constant on each row, diagonals are also equal and probabilities in the last column show the same figures as any diagonal.

As the diagonal associated with the initial generation remains the same as the one of Table 1, we can verify the following equality in the financial equilibrium:

For entrances in T=0

$$(1.05035)^{(-3.911+7.705)} = \frac{1.792 * 10.000 * 0.6}{4.849 * 10.000 * 0.1841}$$

where 5,035% is the decennial technical interest rate associated with that generation, equivalent to a 0,49% annual.

The survival probabilities remain constant for the other generations, in terms of implicit equivalence shown in Table 6. Therefore, an identical equilibrium equation could be associated. Consequently, the technical interest rate is also the same and the following equality regarding interest rates is true: $i^{(0)}= i^{(1)}= i^{(2)}= i^{(3)}=... =i^{(ef-ei)} = 0.05035$.

Additionally, it is possible to calculate the technical interest rate associated with the pay as you go scheme considering the results shown in Table 6. In this case, the annual rate is 0,51%. The difference among the level of the rate associated with the pay as you and the ones for the different generations is not significant, and it is caused by the particular evolution of entries considered in the horizon of the study.

The dynamic reform studied implies permanent changes both in contribution and replacement rates as a way to compensate survival probabilities growth, which also presents persistent changes. We acknowledge, that even though, from a theoretical point of view, this is an appropriate solution, its implementation could be very complex. Nevertheless, this study can provide a guide for solutions which even if do not imply such persistent adjustments on basic parameters, can diminish the magnitude of income reallocation among consecutive generations of the pay as you go scheme.

CONCLUDING REMARKS

In this study we project the financial equilibrium equation of the pay as you go scheme in the long term. We analyze a year when all participants will be members of future generations. Therefore, the short and medium term financial aspects of the scheme are not considered in the study.

In order to carry out this analysis it is necessary to perform very long term demographic projections which provide information on the future evolution of contributors per year.

We estimate contribution and pension levels for that year as their equality for that period allows reaching the financial equilibrium. We assume that the system is financed exclusively by affiliates' contributions and we also assume that management costs are irrelevant.

The classical equilibrium equation is reformulated through the disaggregation of the elements of the demographic relation to estimate the level of the technical interest rate associated with the pay as you go scheme. Therefore, we can conclude that the annual technical interest rate would be equivalent to entrants' average growth rate in a specific period (recovery period).

On the other hand, these results enable us to formulate parametric changes of the scheme in a simple way, taking into account the necessary overall financial equilibrium of the scheme.

The mentioned changes could be divided into two pure categories: on the one hand, those which entail variations to contribution and replacement rates and/or to the basic average retirement salary and on the other hand, those which imply adjustments in retirement age.

In the first case, the basic equation can be transformed so as to identify the different parameters involved and their impact on the financial equilibrium. The different possible options are analyzed, i.e. changes regarding contribution and replacement rates and retirement average basic salary.

It is necessary to carry out new demographic projections of the scheme to evaluate, in an effective way, any increase in the retirement age. These projections would provide information on the new evolution of entrants projected for the whole horizon of the analysis. Taking into account the mentioned information and applying the formulae associated with the financial equilibrium equation of the scheme in the long term would enable us to evaluate the resultant new contribution and/or replacement rates.

Even though, this approach does not consider what is going on now or in the short term, while the long term reform for future generations is taking place, it is advisable to evaluate changes occurring during transition so as to assure certain continuity in the scheme evolution.

It is also important to highlight the incidence of these parametric reforms upon the different generations. Furthermore, we can state that given a contribution and replacement rate for several generations, each generation will have associated different technical interest rates (return). As we assume dynamic mortality rates, there will be unwanted intergenerational reallocations, particularly those from next generations to far future generations.

As we assume that salary mobility and retirement basic salary are the same for all generations, the differences are caused by the permanent changes in survival probabilities (as a direct consequence of improvements in mortality rates).

So as to reduce this type of reallocation we formulate a reform, where both contribution and replacement rates vary along time and according to age, to compensate changes in those probabilities. This way, the technical interest rates associated with both the different generations and the pay as you go scheme will show identical level.

These dynamic changes in the value of the basic parameters imply that the dichotomy between pure defined benefit schemes and defined contribution schemes does no longer exist, as with this kind of adjustments, both contribution and benefit vary depending on the increase of survival probabilities along time.

Therefore, from a theoretical point of view, dynamic parametric changes allow us to keep a constant level of the technical interest rates associated with the different future generations. However, their implementation might be complex, and annual adjustments would not be significant. Nevertheless, this study could provide a guide for further studies searching for non static and phased in solutions to decrease income reallocation among consecutive generations in the pay as you go scheme.

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